Oxford Cambridge and RSA

Wednesday 14 June 2023 – Afternoon

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Time allowed: 2 hours 15 minutes

* 9 9 6 9 4 8 3 1 5 7 *

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Section A (26 marks)

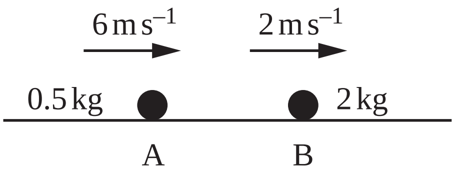
1 A car of mass 800 kg moves in a straight line along a horizontal road.

There is a constant resistance to the motion of the car of magnitude 600 N.

When the car is travelling at a speed of $15\,\mathrm{m\,s^{-1}}$ the power developed by the car is 27 kW.

Determine the acceleration of the car when it is travelling at $15\,\mathrm{m\,s^{-1}}$. **[4]**

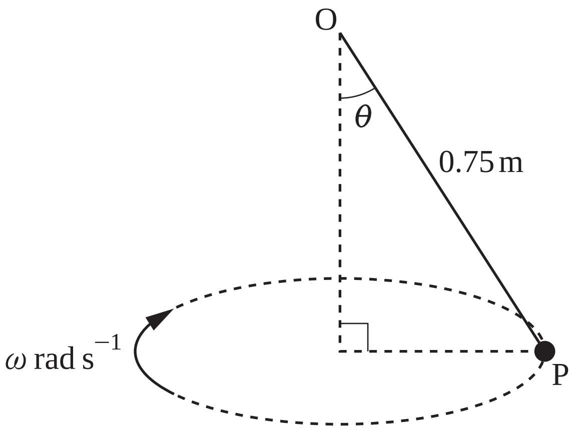
2



Two small uniform smooth spheres A and B have masses 0.5 kg and 2 kg respectively. The two spheres are travelling in the same direction in the same straight line on a smooth horizontal surface. Sphere A is moving towards B with speed $6\,\mathrm{m\,s^{-1}}$ and B is moving away from A with speed $2\,\mathrm{m\,s^{-1}}$ (see diagram). Spheres A and B collide. After this collision A moves with speed $0.2\,\mathrm{m\,s^{-1}}$.

Determine the possible speeds with which B moves after the collision. **[4]**

3



The diagram shows a particle P, of mass 0.2 kg, which is attached by a light inextensible string of length 0.75 m to a fixed point O.

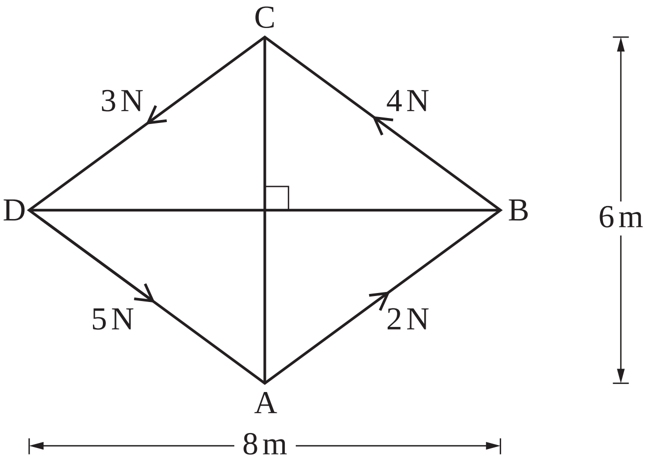
Particle P moves with constant angular speed ω rad s^{-1} in a horizontal circle with centre vertically below O. The string is inclined at an angle θ to the vertical.

The greatest tension that the string can withstand without breaking is 15 N.

(a) Find the greatest possible value of θ, giving your answer to the nearest degree. **[2]**

(b) Determine the greatest possible value of ω. **[3]**

4



A rigid lamina of negligible mass is in the form of a rhombus ABCD, where AC = 6 m and BD = 8 m. Forces of magnitude 2 N, 4 N, 3 N and 5 N act along its sides AB, BC, CD and DA, respectively, as shown in the diagram. A further force **F** N, acting at A, and a couple of magnitude G Nm are also applied to the lamina so that it is in equilibrium.

(a) Determine the magnitude and direction of **F**. **[4]**

(b) Determine the value of G. **[2]**

5 A particle P of mass m kg is projected with speed $u\,\mathrm{m\,s^{-1}}$ along a rough horizontal surface. During the motion of P, a constant frictional force of magnitude F N acts on P. When the velocity of P is $v\,\mathrm{m\,s^{-1}}$, it experiences a force of magnitude kv N due to air resistance, where k is a constant.

(a) Determine the dimensions of k. **[3]**

At time T s after projection P comes to rest. A formula approximating the value of T is

$$T = \frac{mu}{F} - \frac{kmu^2}{2F^2} + \frac{1}{3}k^2 m^{\alpha} u^{\beta} F^{\gamma}.$$

(b) Use dimensional analysis to find α, β and γ. **[4]**

Section B (94 marks)

6 At time t seconds, where $t \geqslant 0$, a particle P has position vector $\mathbf{r}$ metres, where

$$\mathbf{r} = (2t^2 - 12t + 6)\mathbf{i} + (t^3 + 3t^2 - 8t)\mathbf{j}.$$

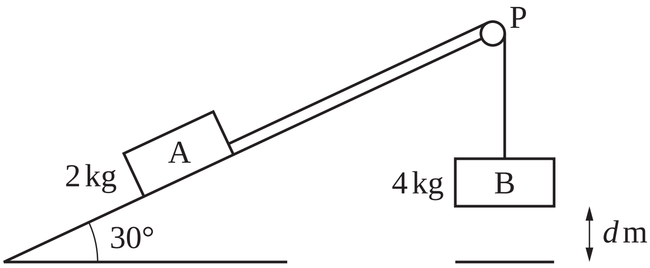
The velocity of P at time t seconds is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

(a) Find $\mathbf{v}$ in terms of t. **[1]**

(b) Determine the speed of P at the instant when it is moving parallel to the vector $\mathbf{i} - 4\mathbf{j}$. **[5]**

(c) Determine the value of t when the magnitude of the acceleration of P is $20.2\,\mathrm{m\,s^{-2}}$. **[3]**

7 One end of a rope is attached to a block A of mass 2 kg. The other end of the rope is attached to a second block B of mass 4 kg. Block A is held at rest on a fixed rough ramp inclined at 30° to the horizontal. The rope is taut and passes over a small smooth pulley P which is fixed at the top of the ramp. The part of the rope from A to P is parallel to a line of greatest slope of the ramp. Block B hangs vertically below P, at a distance d m above the ground, as shown in the diagram.



Block A is more than d m from P. The blocks are released from rest and A moves up the ramp. The coefficient of friction between A and the ramp is $\frac{1}{2\sqrt{3}}$.

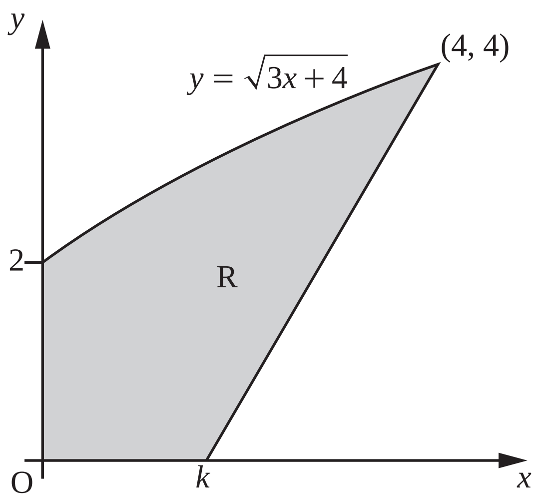
The blocks are modelled as particles, the rope is modelled as light and inextensible, and air resistance can be ignored.

(a) Determine, in terms of g and d, the work done against friction as A moves d m up the ramp. **[3]**

(b) Given that the speed of B immediately before it hits the ground is $1.75\,\mathrm{m\,s^{-1}}$, use the work–energy principle to determine the value of d. **[5]**

(c) Suggest one improvement, apart from including air resistance, that could be made to the model to make it more realistic. **[1]**

8



The diagram shows the shaded region R bounded by the curve $y = \sqrt{3x+4}$, the x-axis, the y-axis, and the straight line that passes through the points $(k, 0)$ and $(4, 4)$, where $0 < k < 4$.

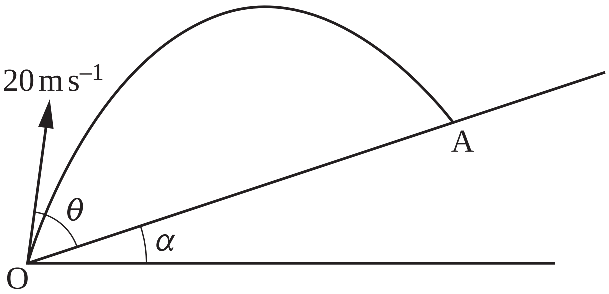
Region R is occupied by a uniform lamina.

(a) Determine, in terms of k, an expression for the y-coordinate of the centre of mass of the lamina. Give your answer in the form $\frac{a+bk}{c+dk}$, where a, b, c and d are integers to be determined. **[6]**

(b) Show that the y-coordinate of the centre of mass of the lamina cannot be $\frac{3}{2}$. **[2]**

9 In this question take $g = 10$.

A small ball P is projected with speed $20\,\text{m}\,\text{s}^{-1}$ at an angle of elevation of $(\alpha + \theta)$ from a point O at the bottom of a smooth plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$ and $\tan\theta = \frac{3}{4}$. The ball subsequently hits the plane at a point A, where OA is a line of greatest slope of the plane, as shown in the diagram.



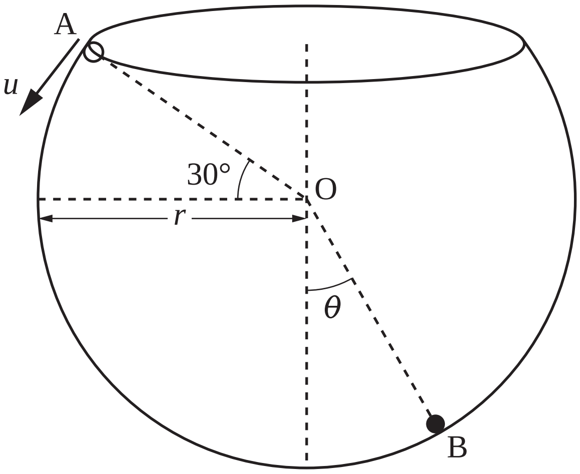
(a) Determine the following, in either order.

- The components of the velocity of P, parallel and perpendicular to the plane, immediately before P hits the plane at A.
- The distance OA. **[9]**

After P hits the plane at A it continues to move away from O. Immediately after hitting the plane at A the direction of motion of P makes an angle β with the horizontal.

(b) Determine the maximum possible value of β, giving your answer to the nearest degree. **[3]**

10



A hollow sphere has centre O and internal radius r. A bowl is formed by removing part of the sphere. The bowl is fixed to a horizontal floor, with its circular rim horizontal and the centre of the rim vertically above O.

The point A lies on the rim of the bowl such that AO makes an angle of 30° with the horizontal (see diagram).

A particle P of mass m is projected from A, with speed u, where $u > \sqrt{\frac{gr}{2}}$, in a direction perpendicular to AO and moves on the smooth inner surface of the bowl.

The motion of P takes place in the vertical plane containing O and A. The particle P passes through a point B on the inner surface, where OB makes an acute angle θ with the vertical.

(a) Determine, in terms of m, g, u, r and θ, the magnitude of the force exerted on P by the bowl when P is at B. **[7]**

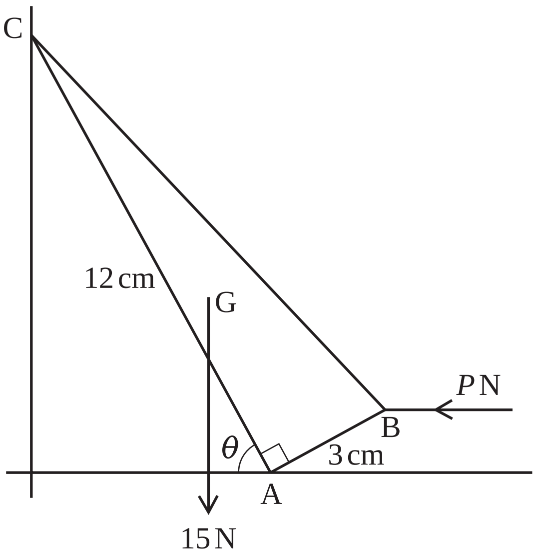
The difference between the magnitudes of the force exerted on P by the bowl when P is at points A and B is $4mg$.

(b) Determine, in terms of r, the vertical distance of B above the floor. **[4]**

It is given that when P leaves the inner surface of the bowl it does not fall back into the bowl.

(c) Show that $u^2 > 2gr$. **[5]**

11



The diagram shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a right-angled triangle ABC, with AB perpendicular to AC, which lies in a vertical plane. The length of AB is 3 cm, and the length of AC is 12 cm.

The prism is resting in equilibrium on a horizontal surface and against a vertical wall. The side AC of the prism makes an angle θ with the horizontal.

A horizontal force of magnitude P N is now applied to the prism at B. This force acts towards the wall in the vertical plane which passes through the centre of mass G of the prism and is perpendicular to the wall.

The weight of the prism is 15 N and the coefficients of friction between the prism and the surface, and between the prism and the wall, are each $\frac{1}{2}$.

(a) Show that the least value of P needed to move the prism is given by

$$P = \frac{40\cos\theta + 95\sin\theta}{16\sin\theta - 13\cos\theta}.$$ **[8]**

(b) Determine the range in which the value of θ must lie. **[4]**

12 Two small uniform smooth spheres A and B are of equal radius and have masses m and λm respectively. The spheres are on a smooth horizontal surface.

Sphere A is moving on the surface with velocity $u_1\mathbf{i}+u_2\mathbf{j}$ towards B, which is at rest. The spheres collide obliquely. When the spheres collide, the line joining their centres is parallel to $\mathbf{i}$.

The coefficient of restitution between A and B is e.

(a) **(i)** Explain why, when the spheres collide, the impulse of A on B is in the direction of $\mathbf{i}$. **[1]**

(ii) Determine this impulse in terms of λ, m, e and u_1. **[6]**

The loss in kinetic energy due to the collision between A and B is $\frac{1}{8}mu_1^{\,2}$.

(b) Determine the range of possible values of λ. **[6]**

13 A particle P of mass m is fixed to one end of a light spring of natural length a and modulus of elasticity man^2, where $n > 0$. The other end of the spring is attached to the ceiling of a lift. The lift is at rest and P is hanging vertically in equilibrium.

(a) Find, in terms of g and n, the extension in the spring. **[3]**

At time $t = 0$ the lift begins to accelerate upwards from rest. At time t, the upward displacement of the lift from its initial position is y and the extension of the spring is x.

(b) Express, in terms of g, n, x and y, the upward displacement of P from its initial position at time t. **[2]**

(c) Given that $\ddot{y} = kt$, where k is a positive constant, express the upward acceleration of P in terms of $\ddot{x}$, k and t. **[1]**

(d) Show that x satisfies the differential equation

$$\ddot{x} + n^2 x = kt + g.$$ **[3]**

(e) Verify that $x = \frac{1}{n^3}(knt + gn - k\sin(nt))$. **[4]**

(f) By considering $\dot{x}$ comment on the motion of P relative to the ceiling of the lift for all times after the lift begins to move. **[2]**

END OF QUESTION PAPER